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teger in our decimal notation is, in theory, followed by a decimal point, and it can make no difference in the meaning of the expression whether the decimal point is expressed or understood. $4.\frac{1}{2}$ must equal 4.5 and not 4.05, which is the equivalent of $4.0\frac{1}{2}$.

$$\text{So } 1.\frac{9}{.001}=1\frac{9}{.001}=1+\frac{9}{.009}.$$

Also solved by *COOPER D. SCHMITT*, *D. A. LEHMAN*, *ELMER SCHUYLER*, and the *PROPOSER*. These contributors agree that the result is $10\frac{890}{901}\%$. To my mind, the solution and discussion of the problem as published above are correct. ED. F.

## ALGEBRA.

**94.** **Proposed by J. W. YOUNG, Columbus, Ohio.**

$$\text{Solve}: \left[\frac{x^2+14x+1}{p^4+14p^2+1}\right]^3=\frac{x(x-1)^4}{p^2(p^2-1)^4}.$$

**I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.**

Let $p^2(p^2-1)^4=A$, $(p^4+14p^2+1)^3=B$.

$$\therefore Ax^6+42Ax^5+591Ax^4+2828Ax^3+591Ax^2+42Ax+A$$
$$=Bx^5-4Bx^4+6Bx^3-4Bx^2+Bx.$$

$$\therefore Ax^6+(42A-B)x^5+(591A+4B)x^4+(2828A-6B)x^3$$
$$+(591A+4B)x^2+(42A-B)x+A=0.$$

$$\therefore A[x^3+(1/x^3)]+(42A-B)[x^2+(1/x^2)]+(591A+4B)[x+(1/x)]$$
$$+(2828A-6B)=0.$$

$$\therefore A[x+(1/x)]^3+(42A-B)[x+(1/x)]^2+(588A+4B)[x+(1/x)]$$
$$+(2744A-4B)=0.$$

Let $[x+(1/x)]=y$.

$$\therefore p^2(p^2-1)^4y^3-(p^{12}+759p^8+2576p^6+759p^4+1)y^2$$
$$+4(p^{12}+189p^{10}+3p^8+3710p^6+3p^4+189p^2+1)y$$
$$-4(p^{12}-644p^{10}+3335p^8-1288p^6+3335p^4-644p^2+1)=0.$$

Let $a=p^2+1/p^2$.

$$\therefore (2-a)^2y^3-(a^3+756a+2576)y^2+4(a^3+189a^2+3332)y$$
$$-4(a^3-644a^2+3332a)=0.$$

$$\therefore (y-a)[(2-a)^2y^2-4(a^2+188a+644)y+4(a^2-644a+3332)]=0.$$

$$\therefore\ y=a \text{ and } y=\frac{2(a^2+188a+644)\pm64\sqrt{(a^3+30a^2+252a+392)}}{(2-a)^2}.$$

Let $p+1/p=b$.

$$\therefore\ y=b^2-2,\ y=\frac{2(b^2+28b+68)}{(b-2)^2},\ y=\frac{2(b^2-28b+68)}{(b+2)^2}.$$

$$\therefore\ p^2x^2-(p^4+1)x+p^2=0.$$

$$(b-2)^2x^2-2(b^2+28b+68)x+(b-2)^2=0.$$

$$(b+2)^2x^2-2(b^2-28b+68)x+(b+2)^2=0.$$

$$\therefore\ x=p^2,\ x=1/p^2,\ x=\frac{(p^4+28p^3+70p^2+28p+1)\pm 8(p^3+7p^2+7p+1)\sqrt{p}}{(p-1)^4}.$$

$$x=\frac{(p^4+28p^3+70p^2-28p+1)\pm8(p^3-7p^2+7p-1)\sqrt{(-p)}}{(p+1)^4}.$$

### II. Solution by ELMER SCHUYLER, Reading, Pa.

Let $p^2=t$, $x^2+1=xz$, and $t^2+1=tv$. Then

$$\frac{x^3(z+14)^3}{t^3(v+14)^3}=\frac{x^2(z-2)^2}{t^3(v-2)^2}.\quad \therefore\left(\frac{z+14}{v+14}\right)^3=\left(\frac{z-2}{v-2}\right)^2.$$

By inspection, $z=v$ fulfills conditions.

$\therefore\dfrac{x^2+1}{x}=\dfrac{p^4+1}{p^2}$, which gives $x=p^2$ or $1/p^2$ (two answers).

Since $\dfrac{(z+14)^3-(v+14)^3}{(v+14)^3}=\dfrac{(z-2)^2-(v-2)^2}{(v-2)^2}$,

we can eliminate, factor $z-v$, and get a quadratic in $z$, or a quadro-quadratic in $x$, which equations give

$$x=\left[\frac{1+\theta\sqrt{p}}{1-\theta\sqrt{p}}\right]^4,\ \theta^4=1.$$

[Quadro-quadratic meaning quadratic equation whose unknown quantity is a quadratic.]

### III. Solution by E. D. ROE, Jr., A. M., Ph. D., Professor of Mathematics, Oberlin College, Oberlin, Ohio.

Put $x=z^2$ and divide numerators by $z^6$, and denominators by $p^6$. We get:

$$\left(\frac{z^2+14+(1/z^2)}{p^2+14+(1/p^2)}\right)^3=\frac{[z-(1/z)]^4}{[p-(1/p)]^4}\ldots\ldots(1).$$

We notice that $z=p$, and $1/p$.

Put $z-1/z=y$, $p-1/p=r$, and (1) becomes

$$\left(\frac{y^2+16}{r^2+16}\right)^3=\left(\frac{y}{r}\right)^4 \ldots\ldots(2).$$

Multiplying out, we have,

$$y^6r^4+3.16y^4r^4+3.16^2y^2r^4+16^3r^4-r^6y^4-3.16r^4y^4-3.16^2r^2y^4-16^3y^4=0, \text{ or}$$

$$r^4y^4(y^2-r^2)-3.16^2r^2y^2(y^2-r^2)--16^3(y^2+r^2)(y^2-r^2)=0,$$

$$\text{or } (y^2-r^2)[r^4y^4-16^2(3r^2+16)y^2-16^3r^2]=0 \ \ldots\ldots(3).$$

From this we have $y^2-r^2=0\ldots\ldots(4)$,

$$\text{or } r^4y^4-16^2(3r^2+16)y^2-16^3r^2=0\ldots\ldots(5).$$

By replacing $y$, and $r$, (4) gives

$$z^2-2+(1/z^2)=x-2+(1/x)=p^2-2+(1/p^2), \text{ or } x+(1/x)=p^2+(1/p^2),$$

$$\text{whence } x^2-[p^2+(1/p^2)]x+1=0, \text{ and } (x-p^2)[x-(1/p^2)]=0,$$

and $x=p^2$, $x=1/p^2$, as before noticed.

Solving (5) for $y^2$, we have

$$y^2=\frac{2.8^2(3r^2+16)\pm 8^2\sqrt{[4(3r^2+16)^2+r^6]}}{r^4}=x-2+1/x, \text{ and } x+1/x=$$

$$=\frac{2(p^2-1)^4+2.8^2[3(p^2-1)^2+16p^2]p^2\pm 8^2p\sqrt{4[3(p^2-1)^2+16p^2]p^2+(p^2-1)^6}}{(p^2-1)^4}$$

The expression under the radical sign is found to be the square of $p^6+15p^4+15p^2+1$. Taking the upper sign and dividing out the factor $(p+1)^4$ from numerator and denominator,

$$x+1/x=\frac{2(p^4+28p^3+70p^2+28p+1)}{(1-p)^4}.$$

Taking the lower sign and similarly dividing out the factor $(p-1)^4$,

$$x+1/x=\frac{2(p^4-28p^3+70p^2-28p+1)}{(1+p)^4}.$$

It is seen that both these values of $x+1/x$ are contained in the formula,

$$\frac{(1+\varepsilon\sqrt{p})^8+(1-\varepsilon\sqrt{p})^8}{(1-\varepsilon^2p)^4},$$

where $\varepsilon$ is a fourth root of unity, as only even powers of $\varepsilon$ and $\sqrt{p}$ occur, and every even power of $\varepsilon$ is $+1$ or $-1$. For $\varepsilon=+1$, or $-1$, we get the first value, and for $\varepsilon=+i$, or $-i$, we get the second value of $x+1/x$, and these are the only values which the formula admits. Thus

$$x+1/x=\frac{(1+\varepsilon\sqrt{p})^8+(1-\varepsilon\sqrt{p})^8}{(1-\varepsilon^2p)^4}=\frac{(1+\varepsilon\sqrt{p})^8+(1-\varepsilon\sqrt{p})^8}{(1-\varepsilon\sqrt{p})^4(1+\varepsilon\sqrt{p})^2}$$

$$=\left(\frac{1+\varepsilon\sqrt{p}}{1-\varepsilon\sqrt{p}}\right)^4+\left(\frac{1+\varepsilon\sqrt{p}}{1+\varepsilon\sqrt{p}}\right)^4=\lambda+1/\lambda$$

if $\lambda$ denote $\left(\frac{1+\varepsilon\sqrt{p}}{1-\varepsilon\sqrt{p}}\right)^4$. Then as before

$$x^2-[\lambda+(1/\lambda)]x+1=(x-\lambda)[x-(1/\lambda)]=0,$$

and $x=\lambda$, or $x=1/\lambda$, but when $\varepsilon$ takes its four values, the group of values represented by $\lambda$, is the same as the group represented by $1/\lambda$, though not corresponding value by value for the same value of $\varepsilon$. The values of $\lambda$ and $1/\lambda$ are the same for 1, and $-1$, $-1$ and 1, $i$ and $-i$, $-i$ and $i$, respectively.

The six values of $x$ are

$$x=p^2,\ x=1/p^2, \text{ and either } x=\left(\frac{1+\varepsilon\sqrt{p}}{1-\varepsilon\sqrt{p}}\right)^4, \text{ or } x=\left(\frac{1-\varepsilon\sqrt{p}}{1+\varepsilon\sqrt{p}}\right)^4$$

for the last four values, or other similar expressions containing $\varepsilon$, which are easily formed.

## GEOMETRY.

**119. Proposed by WILLIAM HOOVER., A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.**

A sphere touches each of two straight lines which are inclined to each other at a right angle but do not meet; show that the locus of its center is an hyperbolic paraboloid.

**I. Solution by F. ANDEREGG, A. M., Professor of Mathematics, Oberlin College, Oberlin, Ohio.**

If the common perpendicular to the two given lines be taken as the $Z$-axis, its length represented by $2c$, its middle point be taken as origin, a plane parallel to the two lines as $xy$-plane, and the projections of the lines on the $xy$-plane as the $X$ and $Y$-axis, respectively, the equations of the lines will be $y=0$, $z=c$, and $x=0$, $z=-c$. The square of the distance of the point $(x', y', z')$, any point in the required locus, from the first line is $y'^2+(c-z')^2$, and from the second